

Translational and Rotational Effect of Earthquake Ground Motion on a Bridge Substructure

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Abstract—In the present paper, the rotational and translational effect of earthquake ground motion for a four span box girder bridge has been thoroughly investigated. This study is motivated by the fact that in many countries the translational and rotational components of earthquake ground motion is not adequately considered in analyzing the overall response of the structures subjected to earthquake ground excitations. Much consideration is given to only the horizontal components of the earthquake ground motion during the response analysis of structures. In the present research work, P waves, SV waves and Rayleigh wave excitations are considered for different angle of incidence. In the present paper, the four span box girder bridge is modeled considering the effects of vertical and rocking components of P, SV and Rayleigh wave excitations. Ground responses namely displacement, velocity and acceleration of the substructures of the bridge have been considered for rotational and translational effects in addition to the horizontal ground motion due to earthquake and wind.

Keywords: Ground motion, Response, Rotational effects, Translational effects.

1. INTRODUCTION

The lack of adequate information about the effects of the rotational components of earthquake ground motion, specially rocking, in the overall response of structures and of bridges to earthquake ground excitations has motivated this study. Since only the horizontal components of earthquake ground motions is given importance during analysis of structures subjected to earthquake excitation. The objective of this work is to investigate these effects on a box girder bridge model. The model is converted to a lumped mass model consisting of a massless column supporting a concentrated mass.

2. METHODOLOGY

The response of a four span box girder bridge subjected to earthquake excitations is studied. Each span is considered to be of equal length of 50m. The box girder depth is considered to be 2.5m. The bridge is supported on both sides by abutments of depth 4m and width 2m along the length. There are three piers in between the two abutments which has a depth of 15m. The girder is supported on hinge over the left abutment. The spans are considered to be discontinuous and

they are supported on both hinge and roller supports over the piers. The girder is supported on roller supports over the right abutment. The piers caps above the piers are considered to be of 4m depth. The piers and the abutments are analyzed for incident P, SV and Rayleigh wave excitations. For SV wave excitations the angles of incidence below the critical angle of incidence are analysed. A program is prepared to obtain the response for each wave excitation in the form of equation of motion which will be second order differential equations. The solution of the differential equations will provide the response of the structure. The input parameters of the earthquake effects will then be modified in the program. The input parameters will be the frequency of excitation and the angle of incidence of the earthquake waves. In case of Rayleigh waves only the frequency parameter will be given importance. The change in the response of the structure by modifying the input parameters of the waves will be studied.

Table 1: Terms used in the equations

Symbol	Quantity	SI UNIT
ϕ_L	Rocking of the left abutment	Radian
ϕ_M	Rocking of the middle pier	Radian
ϕ_R	Rocking of the right abutment	Radian
ω_{n_L}	Natural frequency of the left abutment	Hertz
ω_{n_M}	Natural frequency of the middle pier	Hertz
ω_{n_R}	Natural frequency of the right abutment	Hertz
m_1	Mass on the left abutment	Kilogram
m_2	Mass on the middle piers between the abutments	Kilogram
κ_α	Longitudinal wave number	Radians/meter

U_x^R	Horizontal displacement of ground below the right abutment	meter			Amplitude of the real component of motion in the vertical direction for P wave	
U_x^L	Horizontal displacement of ground below the left abutment	meter		\bar{U}_{yP}	Amplitude of the real component of the rotational motion for P wave	meter
U_x^M	Horizontal displacement of ground below the middle pier	meter		$\bar{\Psi}_P$	Amplitude of the real component of motion in the horizontal direction for SV wave	
U_y^L	Vertical displacement on the left abutment	meter			Amplitude of the real component of motion in the vertical direction for SV wave	Radian
U_y^M	Vertical displacement on the middle piers	meter		\bar{U}_{xS}	Amplitude of the real component of the rotational motion for SV wave	meter
U_y^R	Vertical displacement on the right abutment	meter			Amplitude of the real component of motion in the horizontal direction for Rayleigh wave	
m_L	Mass of the left abutment	Kg		\bar{U}_{yS}	Amplitude of the real component of motion in the vertical direction for Rayleigh wave	meter
m_M	Mass of the middle pier	Kg		$\bar{\Psi}_S$	Amplitude of the real component of rotational motion for Rayleigh wave	
m_R	Mass of the right abutment	Kg			Frequency of excitation of the earthquake waves	Radian
ζ_L	Damping ratio of the left abutment			\bar{U}_{xY}		meter
ζ_M	Damping ration of the middle piers					
ζ_R	Damping ratio of the right abutment			\bar{U}_{yY}		meter
ζ_R	Rotational displacement of ground surface below the left abutment	Radian				
Ψ_L	Rotational displacement of ground surface below the middle pier	Radian		$\bar{\Psi}_Y$		Radian
Ψ_M	Rotational displacement of ground surface below the right abutment	meter				
Ψ_R	Height of the center of gravity of the girder above the left abutment	meter		ω		Hertz
h_L	Height of the center of gravity of the girder above the middle pier	meter				
h_M	Height of the center of gravity of the girder above the right abutment			l_1, l_2	Length of spans of the box girder bridge	meter
h_R					Radius of gyration of the left abutment	meter
ε_L	Factor influencing the fixed based natural frequency of the left abutment				Radius of gyration of the middle piers	meter
ε_M	Factor influencing the fixed based natural frequency of the middle piers				Radius of gyration of the right abutment	meter
ε_R	Factor influencing the fixed based natural frequency of the right abutment				Amplitude of the incident P and SV waves	meter
ε_g	Gravity ratio				Amplitude of the incident Rayleigh waves	meter
τ	ω_{*t}	Radians				
\bar{U}_{xP}	Amplitude of the real component of motion in the horizontal direction for P wave	meter		A_o	Transverse wave numbe	meter

A_1	Wavenumber of Rayleigh wave	meter
K_β	Rocking stiffness of the middle piers	Radians/ meter
	Rocking stiffness of the right abutment	
K_γ	Rocking stiffness of the left abutment	Radians/ meter
	Velocity of P waves	
K_M	Velocity of SV waves	Newton/ Radian
K_R		Newton/ Radian
K_L		
α		meter/sec meter/sec
β		

For the right abutment the equation of motion is defined by the following equation

$$\begin{aligned} \frac{\partial^2 \phi_R}{\partial \tau^2} + 2\left(\frac{\omega_{n_R}}{\omega}\right)\zeta_R \frac{\partial \phi_R}{\partial \tau} + \left(\frac{\omega_{n_R}}{\omega}\right)^2 \phi_R = & \\ \frac{1}{\epsilon_R} \frac{1}{l_1} \frac{\partial^2 U_x^R}{\partial \tau^2} \cos \phi_R + & \\ \frac{1}{\epsilon_R} \left\{ \frac{1}{l_1} \frac{\partial^2 U_y^R}{\partial \tau^2} + \left(\frac{\omega_{n_R}}{\omega}\right)^2 \epsilon_g + \right. & \\ \left. \frac{m}{6m_R} \frac{h_R}{d_R} \left[\frac{2}{l_1} \frac{\partial^2 U_y^{R_2}}{\partial \tau^2} + \frac{1}{l_1} \frac{\partial^2 U_y^{L_2}}{\partial \tau^2} + 3\left(\frac{\omega_{n_R}}{\omega}\right)^2 \epsilon_g \right] \right\} \sin \phi_R + & \\ 2\left(\frac{\omega_{n_R}}{\omega}\right)\zeta_R \frac{\partial \psi^R}{\partial \tau} + \left(\frac{\omega_{n_R}}{\omega}\right)^2 \psi^R & \end{aligned} \quad (1)$$

For the middle piers the equation of motion can be defined by the following equation

$$\begin{aligned} \frac{\partial^2 \phi_M}{\partial \tau^2} + 2\left(\frac{\omega_{n_M}}{\omega}\right)\zeta_M \frac{\partial \phi_M}{\partial \tau} + \left(\frac{\omega_{n_M}}{\omega}\right)^2 \phi_M = & \\ \frac{1}{\epsilon_M} \left(1 + \frac{m_2}{m_M} \frac{h_M}{d_M}\right) \frac{1}{l_1} \frac{\partial^2 U_x^M}{\partial \tau^2} \cos \phi_M + & \\ \frac{1}{\epsilon_M} \left\{ \frac{1}{l_1} \frac{\partial^2 U_y^M}{\partial \tau^2} + \left(\frac{\omega_{n_R}}{\omega}\right)^2 \epsilon_g + \right. & \\ \left. \frac{h_M}{6d_M} \left[\left(\frac{m_1 + m_2}{m_M}\right) \left(\frac{2}{l_1} \frac{\partial^2 U_y^{L_2}}{\partial \tau^2} + 3\left(\frac{\omega_{n_R}}{\omega}\right)^2 \epsilon_g\right) \right. \right. & \\ \left. \left. + \frac{m_1}{m_M} \frac{1}{l_1} \frac{\partial^2 U_y^{L_1}}{\partial \tau^2} + \frac{m_2}{m_M} \frac{1}{l_1} \frac{\partial^2 U_y^{R_2}}{\partial \tau^2} \right] \right\} \sin \phi_M + & \\ 2\left(\frac{\omega_{n_M}}{\omega}\right)\zeta_M \frac{\partial \psi^M}{\partial \tau} + \left(\frac{\omega_{n_M}}{\omega}\right)^2 \psi^M & \end{aligned} \quad (2)$$

The equation of motion of the left abutment can be defined by the following equation

$$\begin{aligned} \frac{\partial^2 \phi_L}{\partial \tau^2} + 2\left(\frac{\omega_{n_L}}{\omega}\right)\zeta_L \frac{\partial \phi_L}{\partial \tau} + \left(\frac{\omega_{n_L}}{\omega}\right)^2 \phi_L = & \\ \frac{1}{\epsilon_L} \left(1 + \frac{m_1}{m_L} \frac{h_L}{d_L}\right) \frac{1}{l_1} \frac{\partial^2 U_y^L}{\partial \tau^2} \cos \phi_L + & \\ \frac{1}{\epsilon_L} \left\{ \frac{1}{l_1} \frac{\partial^2 U_y^L}{\partial \tau^2} + \left(\frac{\omega_{n_R}}{\omega}\right)^2 \epsilon_g + \right. & \\ \left. \frac{m_1 h_L}{6m_L d_L} \left[\frac{1}{l_1} \frac{\partial^2 U_y^{L_2}}{\partial \tau^2} + \frac{2}{l_1} \frac{\partial^2 U_y^L}{\partial \tau^2} + 3\left(\frac{\omega_{n_R}}{\omega}\right)^2 \epsilon_g \right] \right\} \sin \phi_L + & \\ 2\left(\frac{\omega_{n_L}}{\omega}\right)\zeta_L \frac{\partial \psi^L}{\partial \tau} + \left(\frac{\omega_{n_L}}{\omega}\right)^2 \psi^L & \end{aligned} \quad (3)$$

3. NUMERICAL DATA

The bridge substructure is analysed for earthquake excitations of frequency 30 Hz. The angle of incidence that are considered for P and SV waves are 0,15,30 and 45 degrees. Earthquake will be considered for a duration of 30 seconds. The velocity of P wave and SV wave is considered to be 8000 m/sec and 4618. 8376 m/sec respectively. The maximum response is obtained for each angle of incidence corresponding to each of the three different frequencies respectively. For Rayleigh waves the variation of the angle of incidence is not considered.

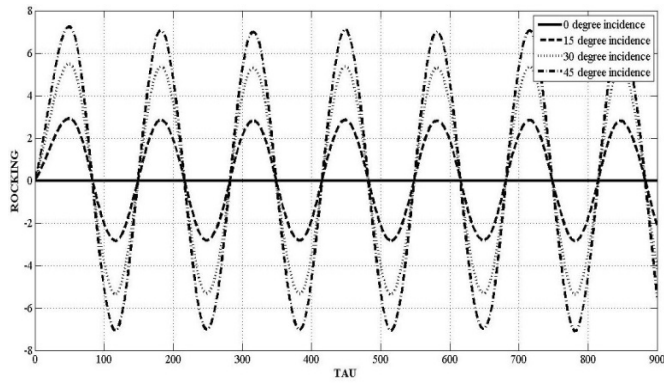


Fig. 1: P waves of 30 Hertz incident on the left abutment

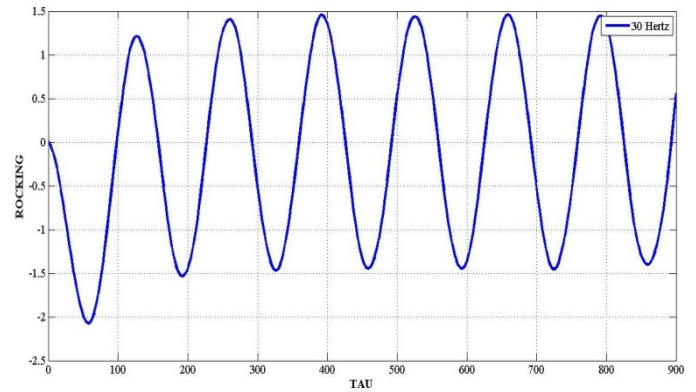


Fig. 5: Rayleigh wave of 30 Hertz incident on the middle piers

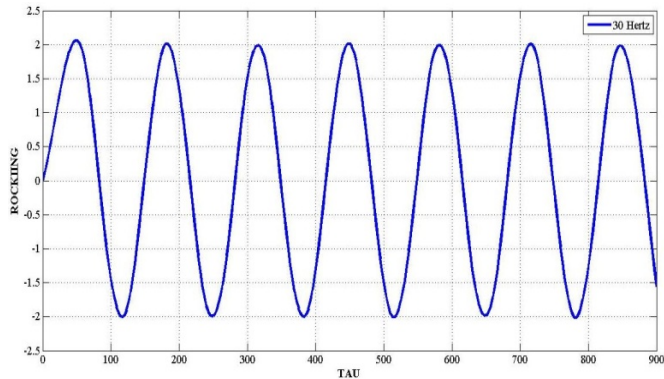


Fig. 2: Rayleigh wave of 30 Hertz incident on the left abutment

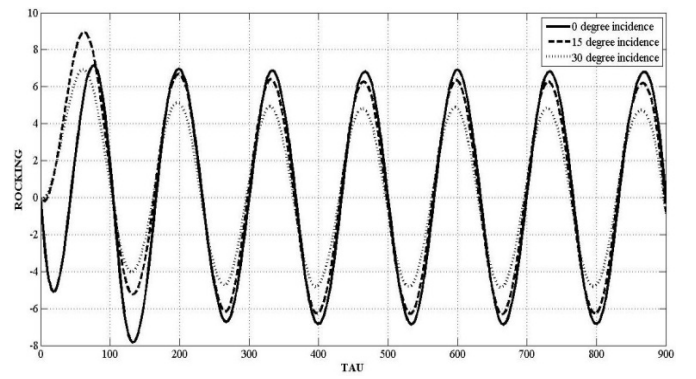


Fig. 6: SV waves of 30 Hertz incident on the middle piers

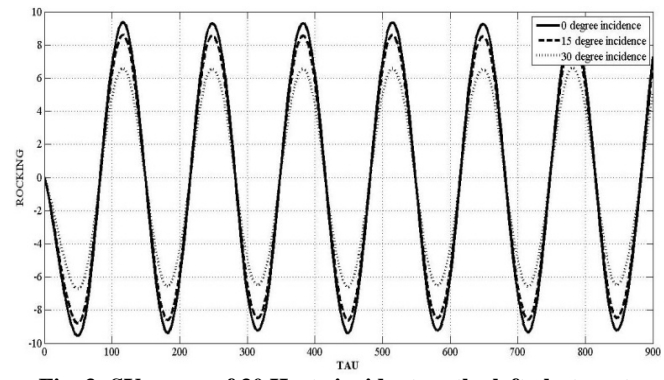


Fig. 3: SV waves of 30 Hertz incident on the left abutment

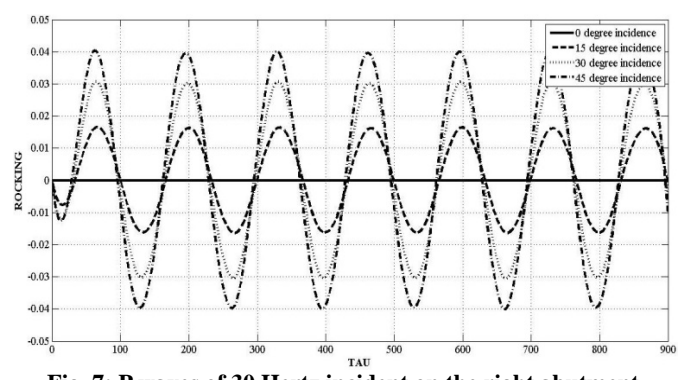


Fig. 7: P waves of 30 Hertz incident on the right abutment

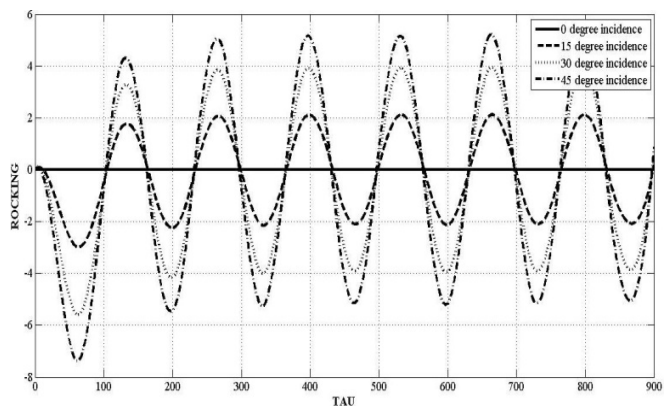


Fig. 4: P waves of 30 Hertz incident on the middle piers

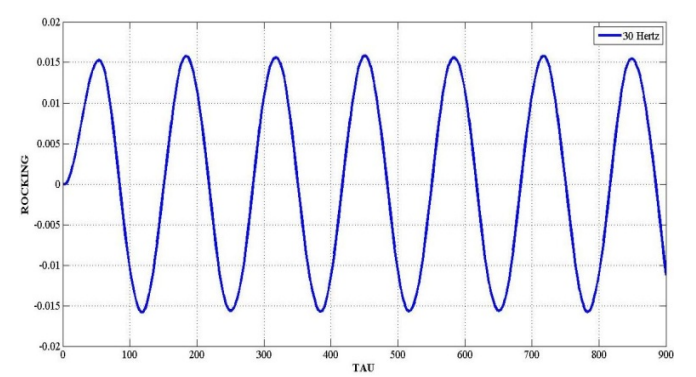


Fig. 8: Rayleigh wave of 30 Hertz incident on right abutment

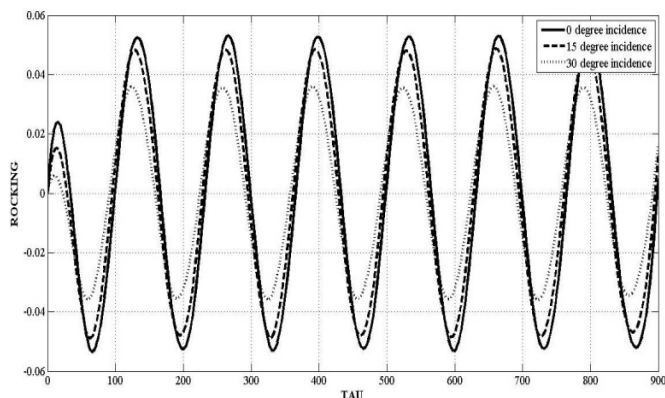


Fig. 9: SV waves of 30 Hertz incident on the right abutment.

4. RESULTS AND DISCUSSIONS

From the graphs of the rocking displacements vs tau that are obtained from solving the second order differential equations we can observe that for P waves incident on the left abutment the rocking displacement increases with increase in the angle of incidence. Since the natural frequency of the left abutment is 31.4376 Hertz it is observed that very high absurd values are obtained for rocking displacement when the frequency of excitation is 30 Hertz. It is due to resonance due to approximately same values of the natural frequency of the system and the frequency of excitation of the earthquake waves.

The natural frequency of the right abutment is calculated to be 133.3169 Hertz and therefore it is observed from the graphs that the rocking displacement for the right abutments are very less for earthquake excitations of frequencies 30 Hertz.

5. CONCLUSIONS

Translational and Rotational components have a significant effect on the structures subjected to earthquake ground motions. In this study it was observed how the magnitude of the rocking displacement was effected by the change in the angle of incidence of the P waves, SV waves and Rayleigh waves. The natural frequency of the system has a major effect in the response of the structure when subjected to earthquake excitation and it has been observed that the system having high natural frequency compared to the excitation frequency of the earthquake undergoes small rocking displacement. Therefore it can be said that a system with higher natural frequency is much suitable for being constructed in zone which is earthquake prone. The natural frequency can be increased by increasing the rocking stiffness of the structure or decreasing either the mass, the radius of gyration or the height of the center of mass of the structure. Concrete of less unit weight can also be used to decrease the mass and thus increasing the natural frequency of the system.

REFERENCES

- [1] Abdel-Ghaffar, A. M. (1977). "Studies on the Effect of Differential Motions of Two Foundations Upon the Response of the Superstructure of a Bridge," California Institute of Technology, Earthquake Engineering Research Laboratory, Report No. EERL-77-02
- [2] Abdel-Ghaffar, A. M. (1984). "Torsional Earthquake Response of Suspension Bridges," J. Engineering Mechanics, ASCE, Vol. 110, 1467-1484
- [3] Achenbach, J. D. (1973). "Wave propagation in Elastic Solids," North-Holland, Amsterdam.
- [4] Blume, J. A., Newmark, N. M. and Corning, H. L. (1961). "Design of Multistorey Reinforced Concrete Buildings for Earthquake Motions," Portland Cement Association, Chicago, 71.
- [5] Beilak, J. D. (1978). "Dynamic Response of Nonlinear Building Foundation Systems," Int. J. Earthquake Eng. And Struct. Dyn., Vol. 6, 17-30.
- [6] Kobori, T. and Shinozaki, Y. (1975). "Torsional Vibration of Structure Due to Obliquely Incident SH waves," Proc. Fifth World Conf. Earthquake Eng., Vol. 1, No. 22
- [7] Luco, J. E. (1976). "Torsional Response of Structures to Obliquely Incident Seismic SH Waves," Earthquake Eng and Struct. Dyn., Vol. 4, 207-219
- [8] Newmark, N. M. (1969). "Torsion in Symmetrical Buildings," Proc. Fourth World Conf. Earthquake Eng., Santiago, Chile, Vol. 2, A-3
- [9] Okamoto, S. (1973). "Introduction to Earthquake Engineering," A Halsted Press Book, John Wiley and Sons, New York.